WHY TEACH MATHEMATICS?

THE AIMS, OUTCOMES AND OPPORTUNITIES AFFORDED BY ITS TEACHING AND LEARNING

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Why teach or learn maths?

What are the intended aims and outcomes of school maths, and the opportunities its teaching affords?

Three groups of aims & outcomes:

A. **Standard aims** or basic reasons for teaching maths

B. **Unintended / unexpected outcomes** of maths - for some students or maybe for all?

C. **Opportunity for visionary aims**
   - What do we wish to see in School Maths?
   - What new emphases would enhance students and society?

A. STANDARD SCHOOL AIMS

1. Basic aim for school maths - Functional numeracy:

- **Ability with maths and numeracy skills adequate for successful general employment and functioning in society**.

This is basic minimal requirement for all at the end of schooling (excluding few with preventative disability).

Mostly achieved by end of primary school

2. Practical, work-related knowledge

- Capability to solve practical problems with maths, especially industry and work-centred problems

  Not necessary for all - depth and type of problems vary across employment types

  Most occupations requiring maths also provide specialist training

3. Advanced specialist knowledge

- ***Not necessary for all adults***
- Needed by minority as foundation for a broad range of university studies, e.g., STEM, medical and social science studies.
- A necessary option in advanced technological society, and more students need encouragement
- Advanced study leads to a highly numerate professional class (e.g., as in France, Hungary)
- **But this aim should not *dominate* or *distort* the maths curriculum for all**

Summary

- **Functional**, **Practical** and **Advanced** knowledge make up useful or necessary maths for some or all
- Primarily for benefit of employment and society from an economic perspective
- Also benefits students in terms of functioning in society, work and further study
- Aim 3 also sustains ***maths itself as a discipline*** *– without specialists maths will not survive*

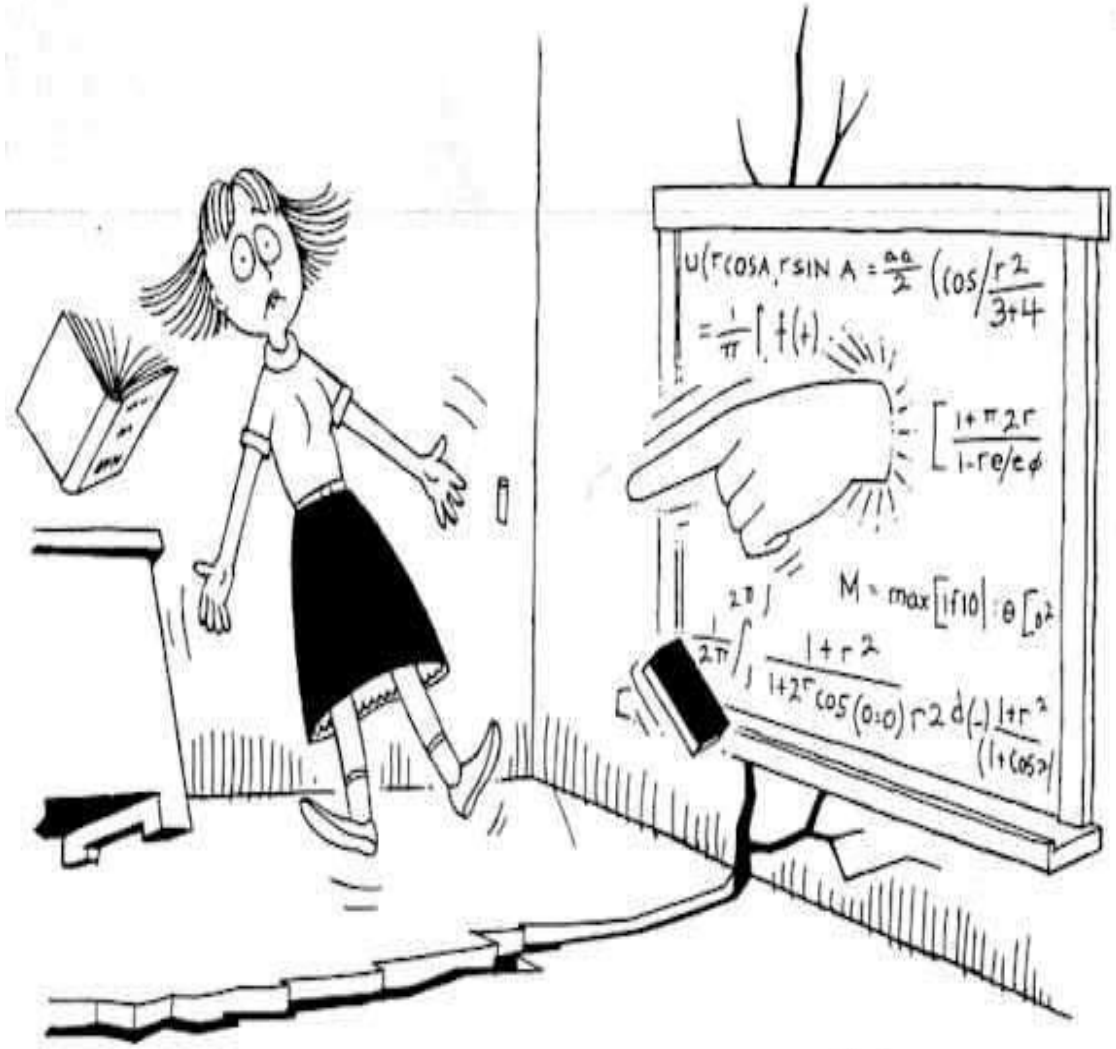
B. UNINTENDED OUTCOMES OF SCHOOL MATHEMATICS

- Unintended outcomes include student **values**, **attitudes** and **beliefs** about **maths** developed during years of schooling
- Not planned or intended but outcomes of the **'hidden' curriculum of schooling**
- Includes beliefs about the ***nature of maths***, about ***what is valuable in maths***, and about ***who can be successful in maths***.
- Examples of these beliefs are:

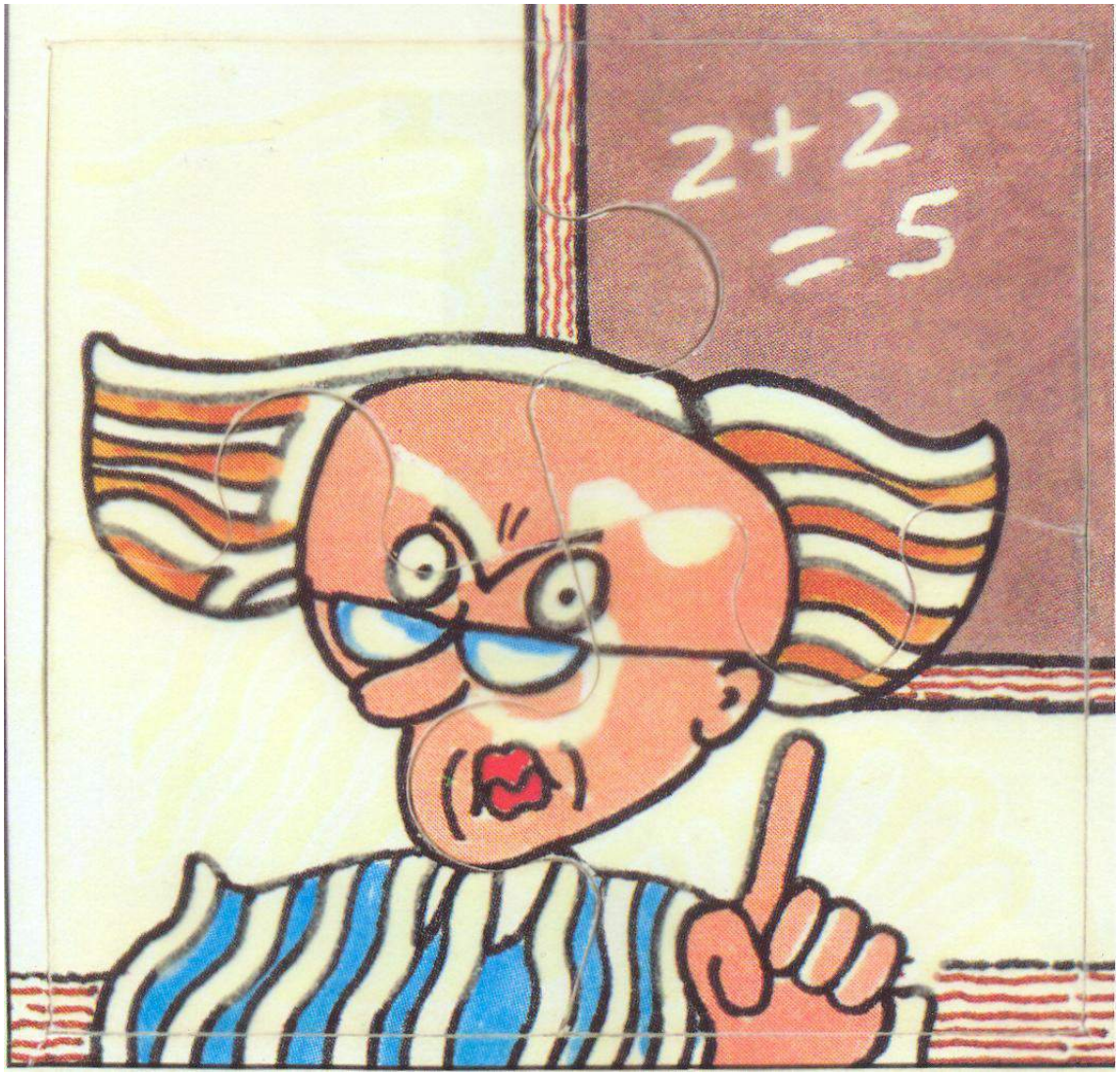
Beliefs about maths include:

- Maths intrinsically **difficult**, inaccessible to most
- Maths success due to **inherited talent**, not effort
- Maths is **male domain**, incompatible with femininity
- Maths is **abstract theoretical subject** disconnected from society and day-to-day life
- Maths is **unchanging and timeless**, completely **objective** and **absolutely certain**
- Maths is **universal**, **value-free** and **culture-free**
- Maths **errors are bad** to be avoided at all cost

ERRORS: TO BE WRONG IS VERY BAD!!



Maths errors and the angry teacher



Maths and the stern, unforgiving teacher

The maths master, by L Haven Hill. *The Windsor Magazine* 1906

Even student teachers fear errors!

Recruits own up to child maths terror

By **Steve Hook**

TEACHING recruits need to overcome their anxiety about maths, according to a study of BEd students.

Eighty per cent of the students, observed at Manchester Metropolitan University, admitted they found maths intimidating when they were at school.

The result is a vicious circle of bad teaching, say the authors of Primary-school teachers' understanding of mathematics and its teaching — published by the Economic and Social Research Council.

A research council spokesman said: "Of the students interviewed, who were all non-maths specialists, the majority had found maths at school intimidating.

"When feelings about maths were explored, some intense emotions surfaced, ranging from pure hatred to, as one student put it, 'a horrific experience'. The students' experience of maths at school had been overwhelmingly negative."

The teaching students said they did not want not to pass their anxieties on to the next generation, although their good intentions were not always realised when they reached school.

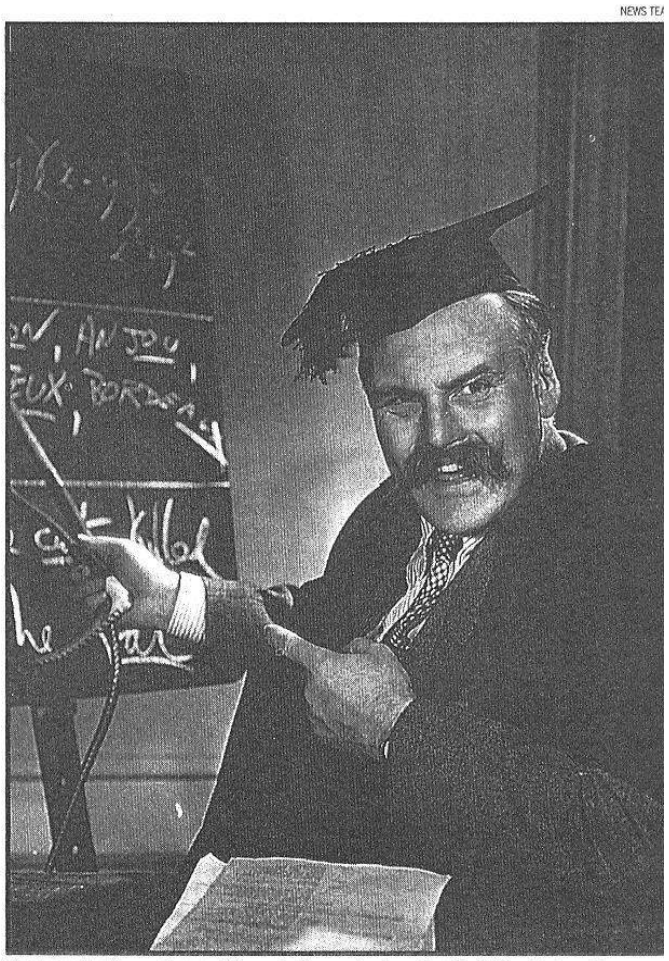
The council said the pressure of the school environment increases students' likelihood of following the patterns of teaching they so loathed when they were pupils. Teaching for the first time, they find themselves under pressure to conform with conventional practice.

As one student said: "It's OK in theory but once you get into a classroom and you are faced with 30 children it's different."

The council said the students remembered having teachers who concentrated on "a version of maths dominated by concern for correct answers which pupils experienced as pressure to get it right or failure in getting it wrong".

The report added: "We confirmed widespread anxieties among our target group concerning their learning of maths and of the prospect of teaching it themselves."

The report is available from the Economic and Social Research Council. Telephone: (01793) 413 122. Fax: (01793) 413 130.



Your number's up: Jimmy Edwards, the caricature head of Whacko!, perhaps represents the stuff of trainee maths teachers' nightmares

ARE THESE IMAGES OUT OF DATE?

NO!

This is primary school child's drawing of a maths teacher – not so long ago

FALSEHOODS

- **Errors are essential steps in learning!**
- **Each of the listed beliefs is *false*!**
- Many of my writings show this, e.g.

  The Philosophy of Mathematics Education, 1991

  Social Constructivism as a Philosophy of Mathematics, 1998
- The good news is that a growing number of researchers and teachers **reject** these beliefs
- Acceptance varies by culture - in Far East maths success seen as ***due to effort not ability***.

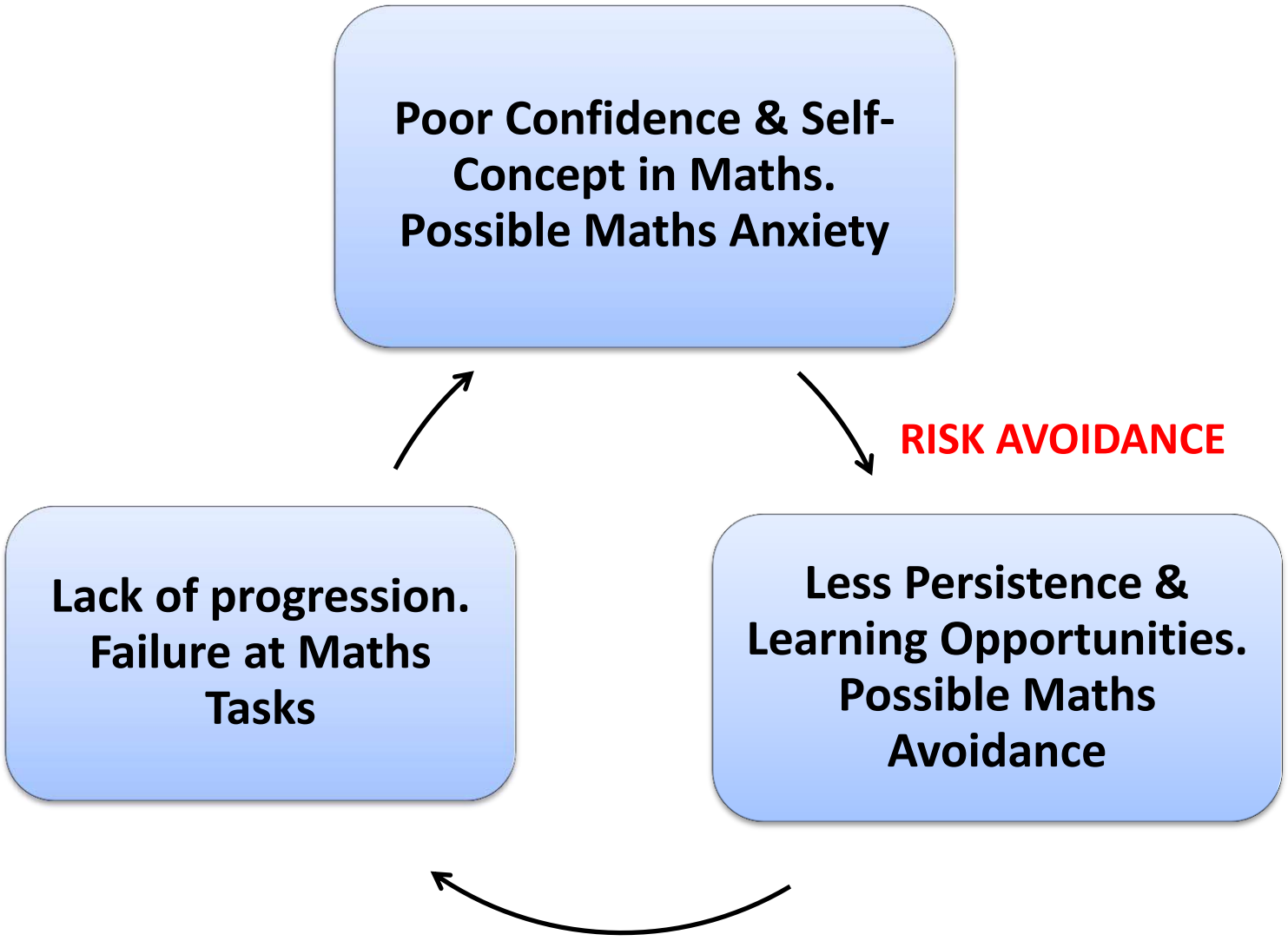
FALSE BELIEFS

- The bad news is such beliefs are **still held by many** students and parents.
- Such false beliefs are spread
  * via popular *images of maths* in media and society
  * through *experience* in some maths classes
- One widespread but not universal outcome - many students develop **negative attitudes** about maths and their own capabilities in maths

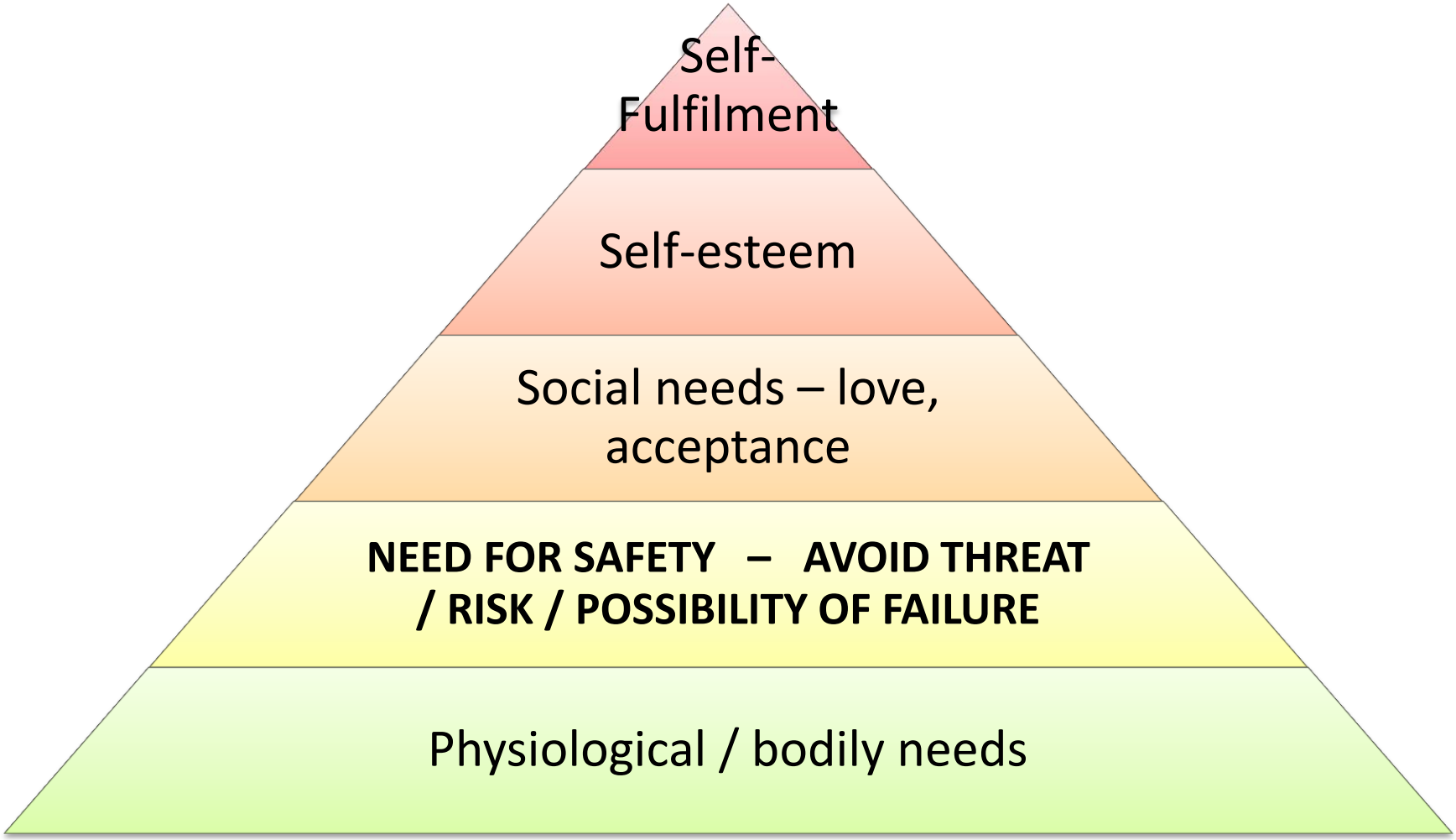
  But as **sports** show, good attitudes are vital for success

  Student lack of confidence in own maths ability – this becomes a self-fulfilling prophecy – **the failure cycle**

The Failure Cycle in Maths



Avoiding risk is rational in the face of threat! (Maslow's Hierarchy of Needs)

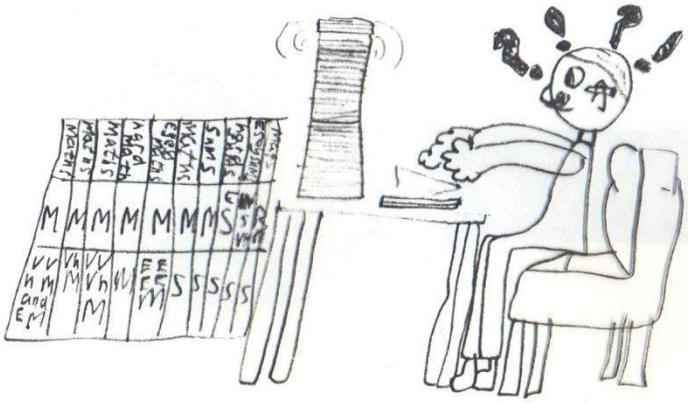


Another False Belief

- Despite progress, mathematics is still widely seen as a **male domain**
- although girls now equal boys in mathematical achievement at 16 years in many countries, and do better in others, too many women still
  - * ***doubt*** *their own abilities,* ***lack*** *confidence*
  - * ***choose not to pursue*** *maths related studies or careers*

Mathematicians seen as male

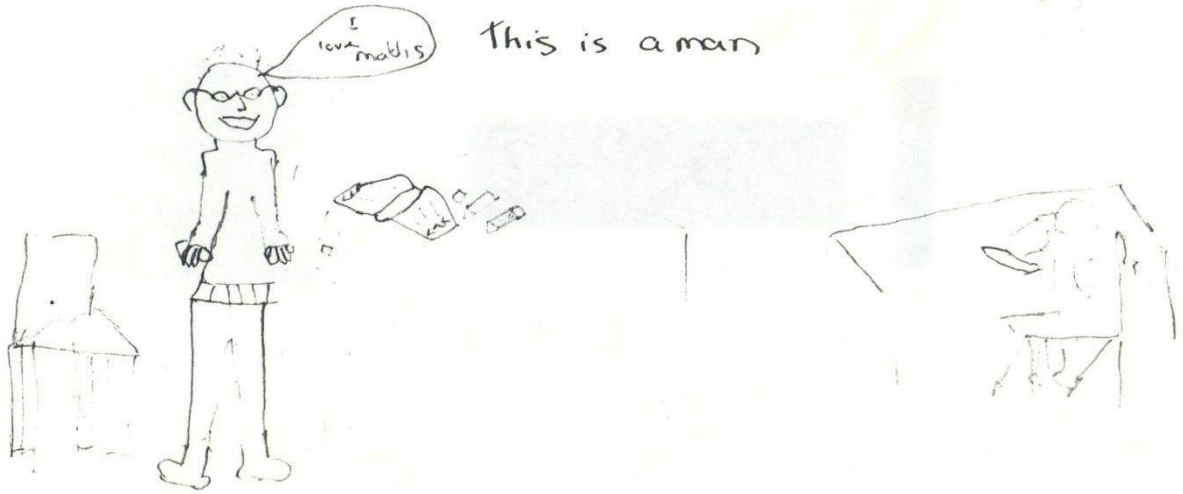
5. Draw a picture of a mathematician.



Boy

He is a male
He is very posh
He makes sums and puts them in to books

5. Draw a picture of a mathematician.



GIRL

This is a man

Beliefs and Inequalities in Maths

Values, images, beliefs & attitudes about maths underlie many differences in learning outcomes across groups defined by

sex
socio-economic status
ethnicity.

E.g., Indigenous mathematics performance in Australia can lag 2+ years behind non-Indigenous students (Queensland Studies Authority, 2004).

But explanation of such inequalities is complex involving social theory, e.g., Bourdieu's cultural capital theory as well as maths beliefs

C. VISIONARY GOALS FOR MATHS

Move from maths curriculum traditionally **defined as content**. Instead propose **empowering** and **broadening aims** for maths

Students should develop:

4. ***Mathematical confidence***
5. ***Creativity through problem posing and solving***
6. ***Critical maths citizenship for social empowerment***
7. ***Broader appreciation of mathematics*.**

These not just ***utilitarian aims*** – but opportunities for more **personal**, **cultural** and **social relevance**

Also have *powerful incidental benefit for society* through developing more **knowledgeable, confident and empowered citizens!**

4. Maths Attitudes & Confidence

- Attitudes are a **vital** and **underestimated** incidental outcome of school maths
- Attitudes uniquely involve development of whole person in a rounded way: **intellect** *and* **feelings**.
- Effective maths knowledge & capabilities rest on
  - * *Feelings of maths enablement and empowerment*
  - * *Confidence to learn and apply maths*
  - * *Enjoyment in learning and using maths*
  - * *Freedom from negative attitudes to maths*

Mathematical Confidence

Maths confidence includes

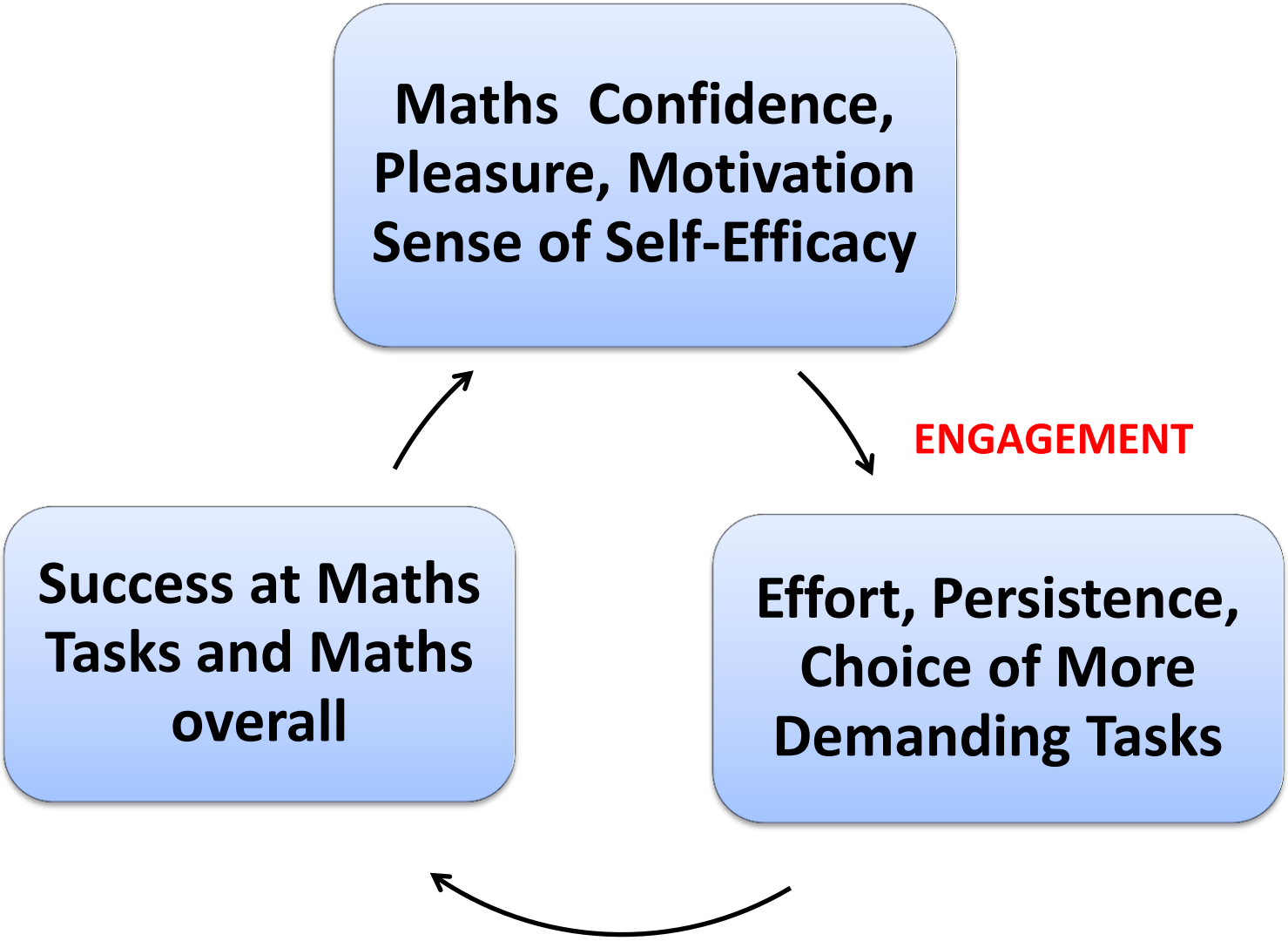
** being confident in one's personal knowledge of maths*
** feeling able to use and apply maths widely*
** confidence in acquiring of new knowledge and skills when needed*

Such confidence leads to

I Persistence in solving difficult mathematical problems,
II Willingness to accept challenging tasks

This is reflected in the virtuous and upwardly spiralling Success Cycle

The Success Cycle in Maths



Building Success

- The success cycle is an intrinsic mechanism drawing us to engagement and effort powered by the pleasures of success and self-enhancement - *like a light draws a moth*.
- We can turn a failure cycle into a success cycle by ***subtracting threat*** and ***making success achievable***
- Done right this should lead to ***positive attitudes***
- In school this means reducing the **dominance of exams** and improving quality of student learning experience (more choice, interest, effort, success)

5. Problem Posing and Solving

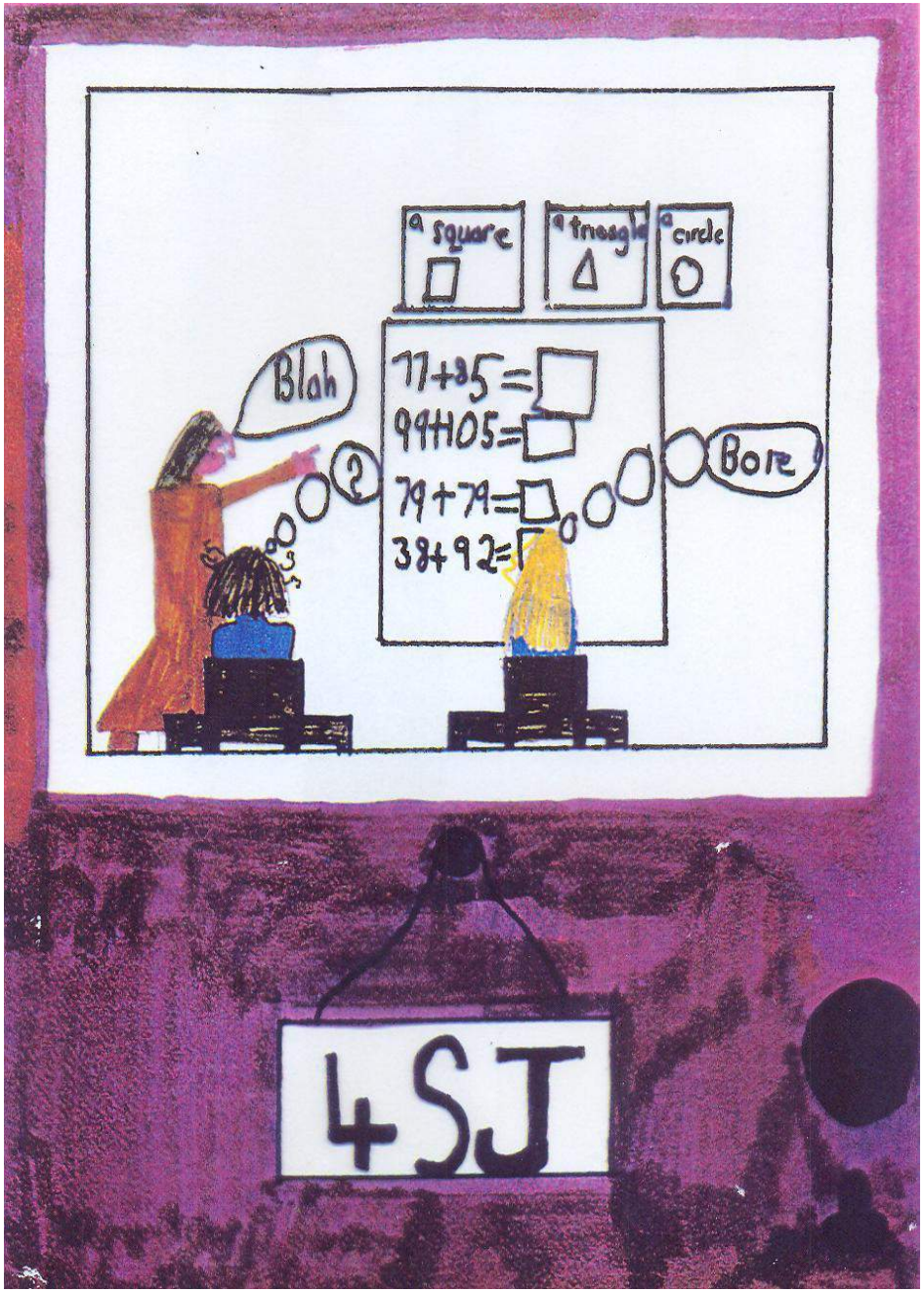
- Maths too often seen as **non-creative** and **mechanical**
- Problem solving too focussed on ***routine problems***
- Great potential for creativity in school maths, but it lies in solving ***non-routine*** problems
- True problem solving requires new methods and approaches to be created for non-routine solutions
- Students **choose** what models and approaches to use in their solutions
- They **create** and apply heuristics – strategic solution methods

MOVE AWAY FROM

1. Passive students

2. Closed maths classroom

3. Focus on unique right answers

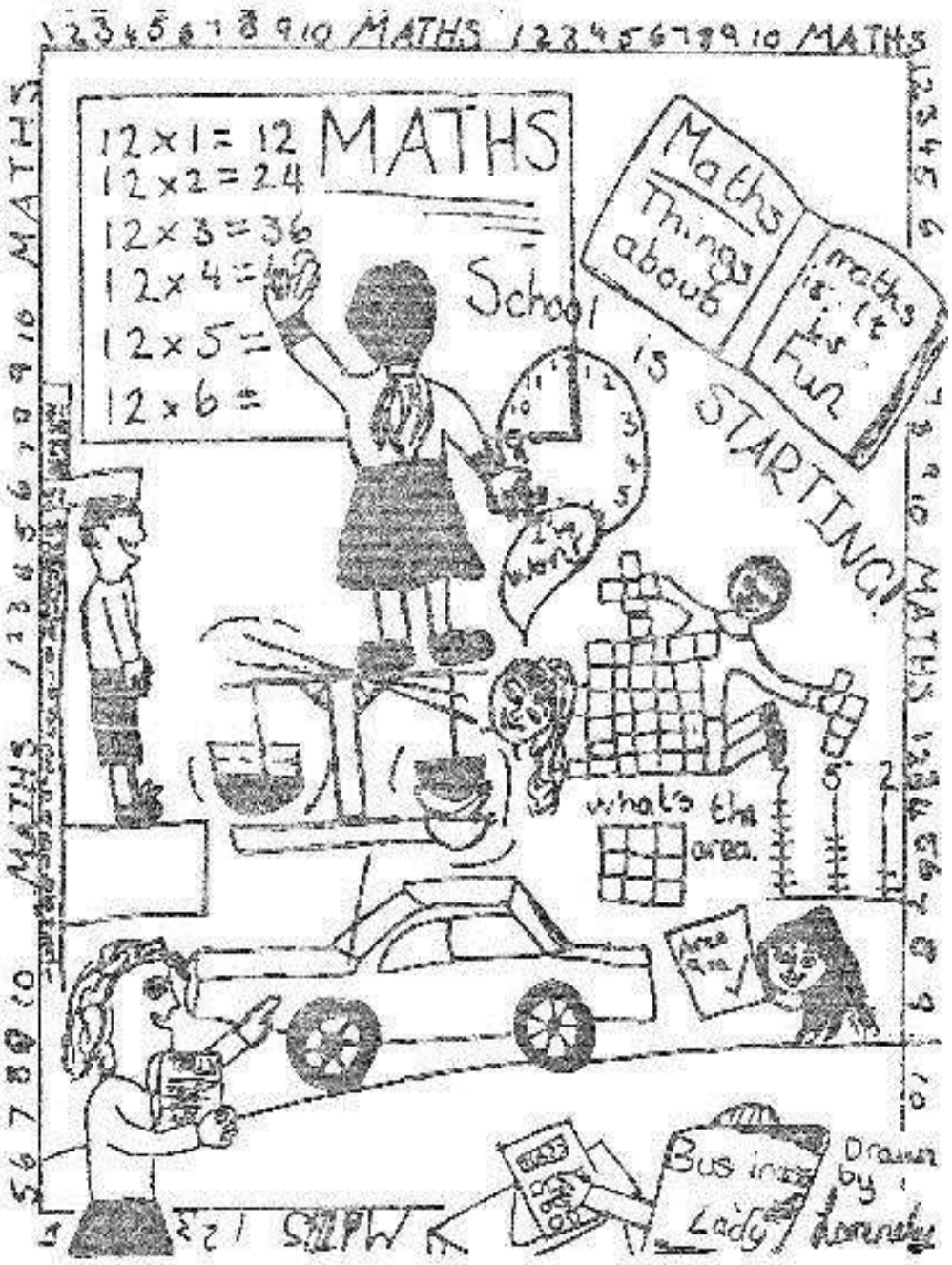


GOAL

1. Active learners

2. Open, creative maths classroom

3. Risk taking encouraged



Problem Posing

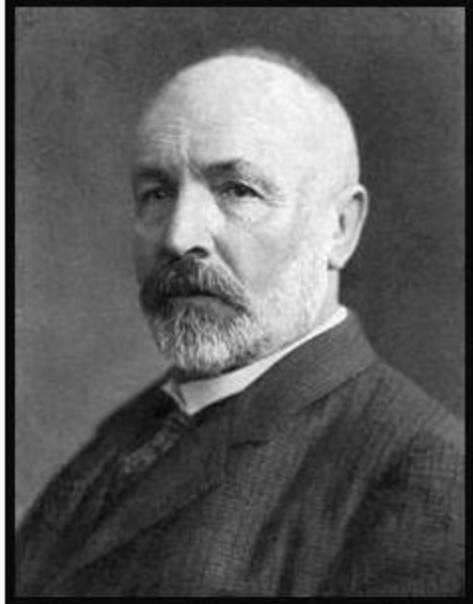
Problem posing is the formulation of **new maths questions** and **problems** to be solved

Problem posing involves students:

* *Choosing aspects of life to probe, model and explain*
* *Choosing mathematical patterns to investigate*
* *Framing questions by generalization, analogy*, etc

Full **creativity** flowers through student choice at each stage of problem solving cycle (Polya)

1. *Problem or model formulation*
2. *The choice of methods to apply*
3. *The construction of solutions*
4. *Deciding which solutions are optimal*



In mathematics the art of proposing a question must be held of higher value than solving it.

(Georg Cantor)

izquotes.com

6. Social Empowerment via Maths

- ***Mathematics is a political subject!***

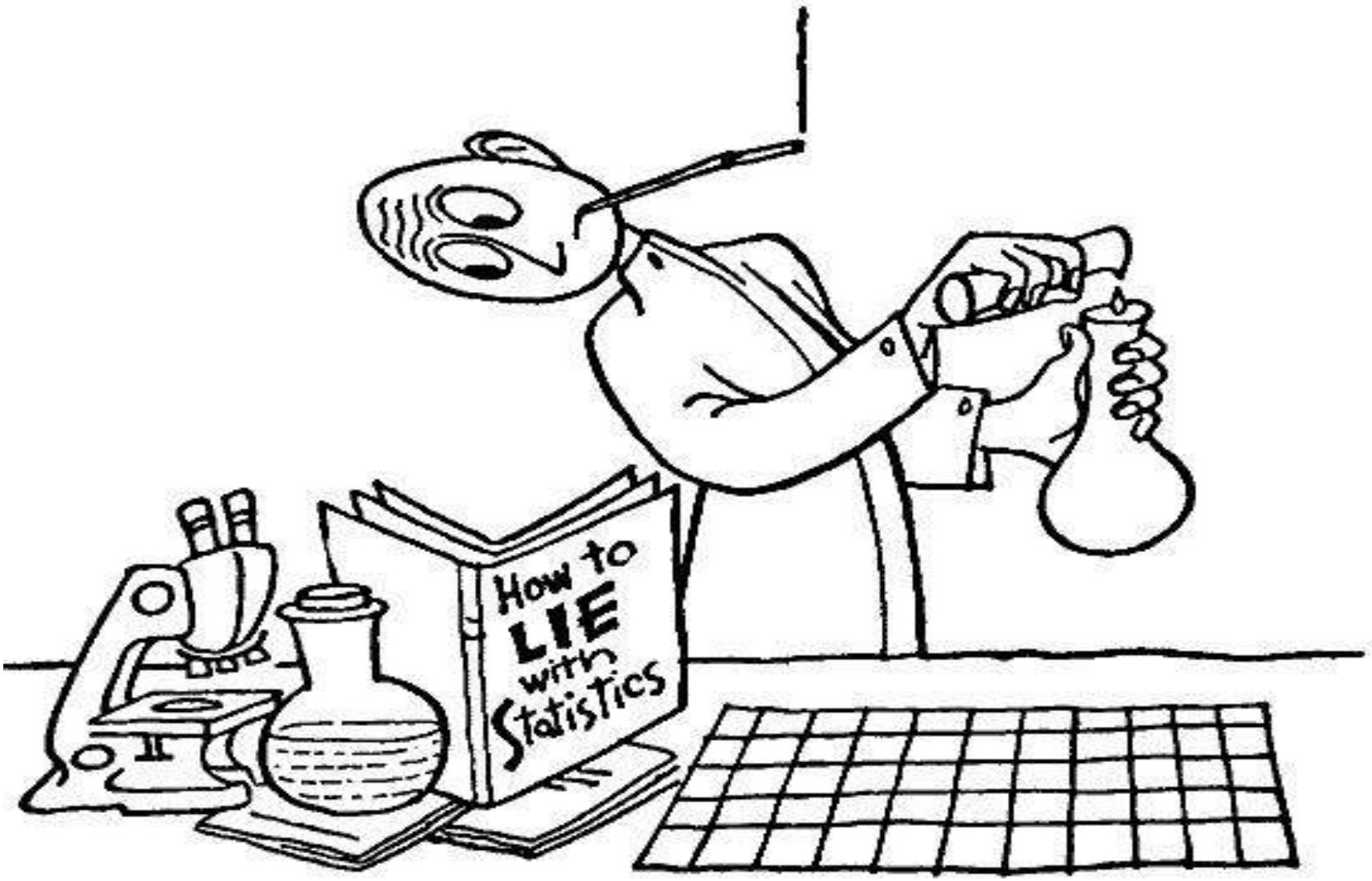
  Maths should socially and politically empower students to be numerate critical citizens in society.

- **Critical mathematical citizenship** involves

  * *Ability to use maths in social and political activity, for betterment of students and democratic society as a whole*
  * *Critically understanding uses of mathematics in society*
  * *Interpreting and critiquing maths in social, commercial and political claims in adverts, headlines, blogs, reports, etc*
  * *Scrutinizing financial sector and government systems and procedures*
  * *Understanding limits of validity of uses of maths, what decisions are concealed, and rejecting spurious or misleading claims.*

- **Every citizen needs these capabilities to defend democracy and the values of humanistic and civilised society**

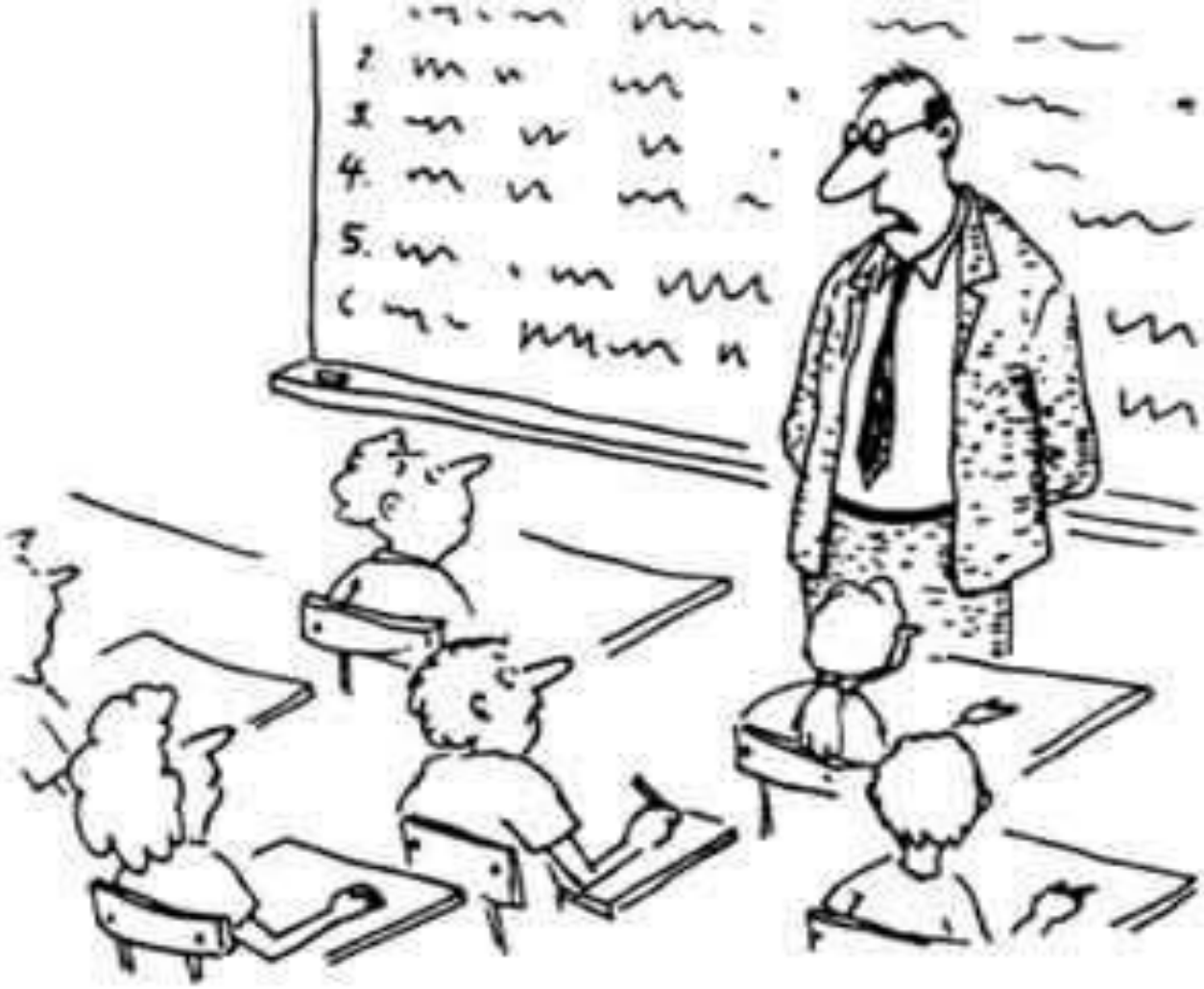
Students must question uses of maths



Critical Mathematical Pedagogy

- To develop critical mathematical literacy and citizenship need to have:
  - *Extended **social maths projects***
  - *Students to **choose** topics that matter to them (environment, aid, social problems, etc)*
  - *Students **choose** what maths methods best suit topics*
  - *Students to **discuss** methods and results in depth together and with teacher*
  - *Classes to **discuss and critique** uses of maths in media – e.g. newspapers*

We need a pedagogy of questions!



"I expect you all to be independent, innovative, critical thinkers who will do exactly as I say!"

7. Appreciation of Mathematics

Language (English, Indonesian) has

- *Reading and writing (language skills & capabilities)*
- *Literature (Appreciating texts and their social and cultural place)*

Maths has

- *Calculating, Solving problems (maths skills & capabilities)*
- *Appreciating maths and its social and cultural place is **missing from school!***

Maths is not only about doing!

Where is the missing *appreciation* aspect of maths?

What is Appreciation of Maths?

Maths **more than** calculating, solving and proving

School maths needs a broader appreciation of

- *Maths in culture, art and social life*
- *History of mathematics and maths in history*
- *Mathematics as a unique discipline*
- *Proof and how maths knowledge validated*
- *Limits of validity of maths applications*
- *Controversies in philosophy of maths*
- *Big ideas of mathematics*

Maths in Culture, Art and Social Life

Students need to appreciate important role of maths in culture, art and society - including:

* ***Symmetry*** *in art and religious symbolism and in our appreciation of beauty*
* ***Algebraic equations*** *in scientific theories, e.g., Einstein's* $E=mc^2$ *underpins modern physics and cosmology*
* *Maths in* ***all aspects of daily life****, e.g., via commerce, economics, stock market, telecommunications, ICT*
* *Maths in representing, coding and displaying* ***all information***

However, need to recognise that maths is becoming invisible - hidden within the programmes and social systems that control complex technological society

History and Maths

- History makes school maths with its abstract concepts meaningful
- **But** there is a myth that *applied maths* arises from practical applications of *pure maths theory*
- **Actually** *maths is first 'applied'* – only later mathematicians abstract and purify concepts and systematize theories
- Maths begins with ***ethnomathematics*** - informal culturally embedded maths concepts and skills found in every culture, past and present, rural and urban
- Once invented some mathematicians study for **its own sake** - *purely out of interest* – e.g., Geometry in Greece, Algebra in Mid East, Number patterns in India, China and Europe
- But social needs in history have driven invention of most mathematical concepts, symbols, methods and problems.

Maths Invented for Social Needs

Country	Social Practice	Maths Topics Developed
Mesopotamia, India, Egypt	Tax, Accounting, Commerce	Numbers and Number Systems, Measures
Egypt	Surveying	Geometry
Medieval Europe & Mid East	Commerce, Navigation, Ballistics	Number Algorithms, Trigonometry, Mechanics
19th Century Europe	Agriculture, Medicine, Insurance	Statistics
Modern Europe	Mechanisation, Cryptography	Logic, Coding Theory

Maths as a Discipline

- Maths should be appreciated as a **distinctive** and **unique discipline**, with its own:
  - *Central branches and topics*
  - *Key concepts linking branches (see Big Ideas)*
  - *Powerful language – basic to other disciplines*
  - *Unique epistemology of maths – maths knowledge validated by deductive proof.*
  - *But proof has its limits – one case disproves, but finite number of cases never proves a generality!*
- Need to know that much ***more to maths*** than **number** and just what is **taught in school**

Philosophy of Maths

- It is a ***shock*** to learn there are controversies in maths, such as over these questions:
  - ***Are the objects of maths discovered or invented?***
  - ***Is maths knowledge certain beyond all doubt?***
  - ***Did maths exist before people evolved?***
- **No right answers here** – although absolutist & superhuman views of maths can make some learners feel excluded
- Why not let students be **excited & stirred** by these ideas and discuss them?
- Discussing these controversies develops a more **critical attitude** to maths, and helps to counter automatic **attributions of certainty** to anything mathematical.

Big Ideas of Maths

Learners should meet and *intuitively understand* of big ideas in maths such as:

pattern, modelling, symmetry, structure, equivalence (invariance), proof, paradox, recursion, randomness, chaos, infinity.

Maths has many of the deepest, most exciting ideas created by humankind.

These fire our imagination - *the scientific equivalent of poetry* - offering noble, aesthetic, and even spiritual experiences

Are these ideas suitable for school?

Are aims about appreciation feasible for school? Are the concepts too complex?

No! - not for firing the **imagination**. Even big ideas like infinity – 9 year olds happily discuss infinity of space and numbers

Maths too often seen as dull, routine, uninspiring

With c. 2000 hours of school maths time we can afford to:

- Develop *appreciation* to inspire and excite
- Grow *critical maths literacy* and *citizenship* for fairer society
- Create expert *problem posers and solvers*
- Develop *confident learners* and users of maths

Such VISIONARY AIMS can build more knowing confident students & citizens, and possibly a better society for all!

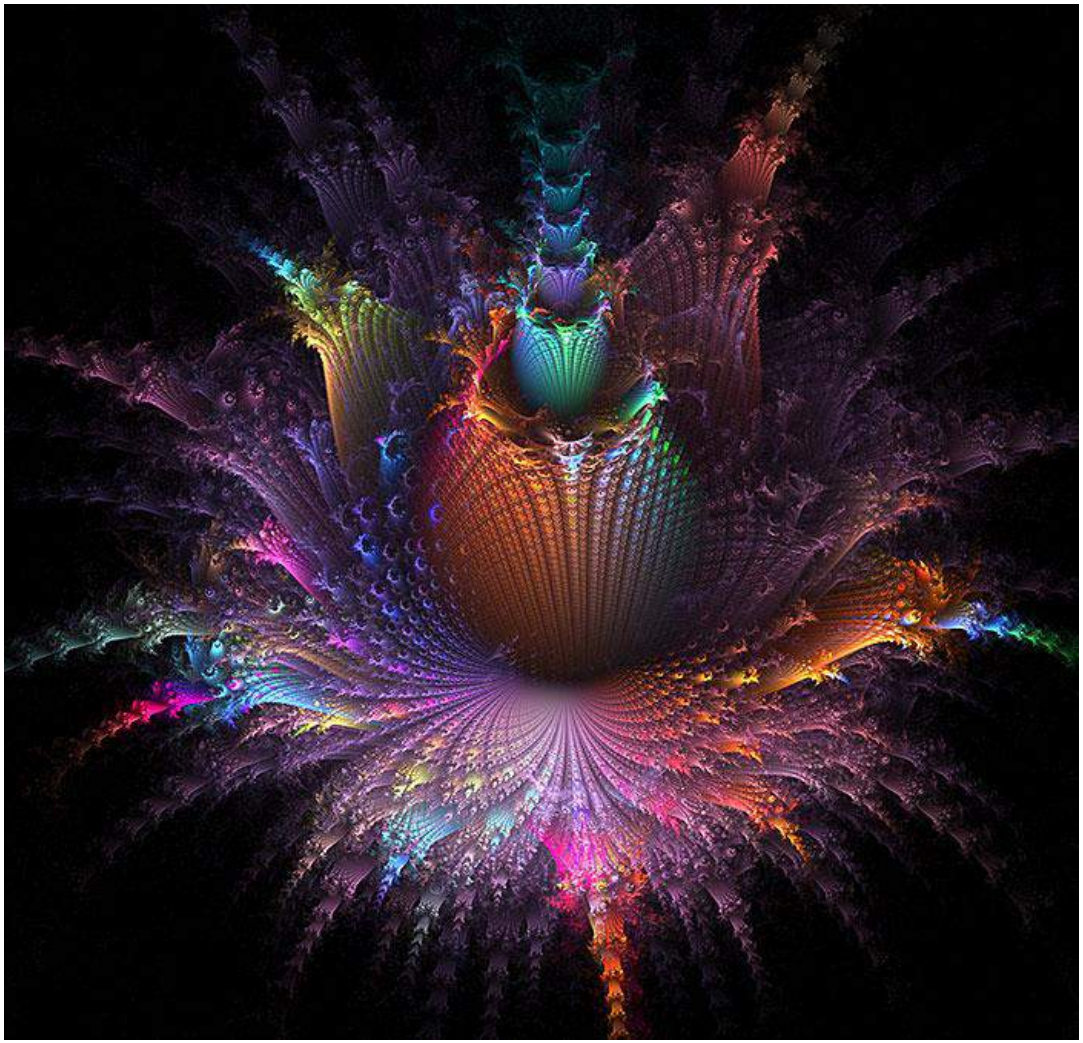
Conclusion

Three groups of aims & outcomes for school maths:

1. **Standard aims** for maths – yes, of course these are needed! – but **Visionary** aims help achieve them too!
2. **The Negative Unintended outcomes** of maths – we must do what we can to **avoid them**
3. **Visionary aims –** ***We must make maths exciting, richly linked to other subjects, empowering, truly educational, fascinating and inspirational!***

You/We can do it!!

The beauty of mathematics!



End of Slideshow

Thanks for listening!

Any Questions, Comments or Reactions?